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Modeling of Dynamic Behavior of Conditional Variancein Crude Oil Prices: The BEKK and CCC-GARCH Frameworks

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ABSTRACT
This study investigates the dynamic behavior of conditional variance and covariance among benchmark crude oil prices; Brent, Dubai and West Texas Intermediate using multivariate GARCH frameworks of Diagonal BEKK-GARCH and Constant Conditional Correlation (CCC-GARCH) models. The data used in this study were obtained from US Energy Information Administration (EIA). The data set consists of weekly prices of three major oil benchmarks: Brent crude oil, Dubai crude oil, and West Texas Intermediate. The Diagonal BEKK-GARCH results reveal statistically significant short-term responses to shocks and strong persistence in volatility, highlighting the clustering effect of typical crude oil markets. Positive definiteness in the BEKK model's covariance matrix confirms model stability and appropriate specification. The CCC-GARCH model identifies sustained volatility with near-unit root behavior and confirms strong static correlation between Brent and WTI, while showing moderate interdependence with Dubai/Oman. However, the CCC-GARCH model offers deeper insights by capturing the time-varying nature of variances, if the correlation between series remains constant over time. The findings reveal major implications for financial risk management, energy market forecasting, and macroeconomic policy. The study emphasizes the need for investors to adopt adaptive hedging techniques and recommends the use of dynamic correlation models for better prediction of volatility and asset co-movements. Policymakers in oil-exporting nations can leverage these insights to inform the design of stabilization mechanisms, sovereign wealth strategies, and counter-cyclical fiscal interventions. The study provides a robust empirical foundation for financial and policy decisions in volatile global energy markets.

Keywords: *Dynamic Behavior, Crude Oil Prices, Volatility, Persistence, MGARCH models*

INTRODUCTION

Crude oil remains one of the most critical commodities in the global economy, serving as an important input in industrial production, energy production, and international trade. The prices of crude oil are measured with respect to Brent, West Texas Intermediate (WTI) and Dubai/Oman. They are central to the pricing of petroleum products globally and are closely monitored by investors, policymakers and researchers. However, these prices are known to exhibit strong volatility, cluster behavior, and interdependencies driven by a mix of geopolitical tensions, supply-demand imbalances, financial speculation, and macroeconomic shocks. Modeling crude oil price volatility is essential for understanding market dynamics, managing investment risks, and formulating macroeconomic policies in oil-exporting countries. Over the past few decades, researchers have used various GARCH models to analyze volatility in the financial time series. The multivariate GARCH (MGARCH) framework, particularly the BEKK-GARCH and Constant Conditional Correlation (CCC-GARCH) models, has been widely used to model the common dynamics of multiple time series by capturing both the volatility of individual assets and the interdependencies between them (Angels & Crowns, 1995; Bollerslev, 1990; Tse & Tsui, 2002). These models provide critical insights into the evolution of conditional variance and correlation across markets, making them useful for risk forecasting and portfolio management.

Despite the robustness of the MGARCH models, several issues are still underexplored in the existing literature. First, while many studies focus on volatility estimation, few explicitly verify the positivedefiniteness of the constant covariance matrix in the BEKK-GARCH specification—a key requirement for model validity and numerical stability. Positive definiteness ensures that the estimated variance-covariance matrices are invertible and meaningful in a financial context (Engle & Kroner, 1995). Second, the static assumption in the CCC-GARCH model may not sufficiently reflect the dynamic interaction and shock transmission between oil price benchmarks, duringa particular periodduring financial turmoil or oil supply disruptions. Failure to account for time-varying correlations can result in inaccurate predictions and suboptimal risk strategies (Silvennoinen&Teräsvirta, 2009).

The motivation for this study stems from the increasing need for robust models that capture volatility persistence, shock sensitivity, and time-varying correlations for major oil benchmarks, especially in an era of high-frequency trading and macroeconomic uncertainty. Oil-dependent economies face significant fiscal vulnerabilities due to volatile crude oil prices, underscoring the importance of developing accurate models that support risk assessment, policy intervention, and sovereign wealth fund planning.

While previous studies have made important contributions to literature, they often suffer from regional focus limitations or exclude certain oil benchmarks. For example, studies by Hamzaoui, &Regaieg, (2016) andKocoglu, et al. (2023) focus heavily on Brent and WTI and overlook the Dubai/Oman benchmark that plays a crucial role in Asian oil markets. Moreover, empirical work by Ali, et al. (2022) and Mollakarimi et al. (2024) largely assumes constant correlations, thereby ignoring the structural fractures and dynamic interdependencies that characterize global energy markets. To address these gaps, this study investigates the dynamics of conditional variance and correlation structures among global crude oil price benchmarks Brent, West Texas Intermediate (WTI) and Dubai/Oman. These multivariate GARCH models are used with the intention of understanding volatility persistence, verifying positive definiteness of the constant covariance matrix in the BEKK-GARCH model and to ensure statistical validity, modelstability, shock sensitivity and time-varying correlations of CCC-GARACH. The study also seeks to identify the implications of time-varying correlations for portfolio diversification, risk management, and macroeconomic policy in oil-dependent economies. By applying the BEKK and CCC-GARCH frameworks, this research provides deeper empirical insights into the volatility structure of global crude oil markets and contributes to improving risk modeling practices in both political and financial time series domains.This research is very important for understanding the behaviour of global crude oil prices, especially given the increasing market volatility driven by geopolitical tensions, policy changes and fluctuations in demand. By comparing the two multivariate GARCH frameworks, the study provides a nuanced understanding of variance behavior and correlation dynamics in key global oil price indicators. It addresses critical shortcomings in traditional constant correlation models by highlighting changes in the relationship between oil price benchmarks and offers a more accurate approach to modelling systemic risks and volatility transfers

METHODOLOGY

The data used in this study were obtained from the U.S. Energy Information Administration (EIA) website [http://www.eia.gov/dnav/pet/pet_stoc_wstk_dcu_nus_w.htm]. The data set consists of weekly prices of three major oil benchmarks: Brent crude oil (COB), Dubai crude oil (COD), and West Texas Intermediate crude oil (COWTI). These prices are expressed in US dollars per barrel from 5 January 1990 to 21 June 2024, representing a total of 1,799 weekly data points. The study used EViews 10 for data analysis. EViews is widely used in econometrics for performing statistical modeling, hypothesis testing, and generating graphical results. Preliminary analysis of the data is performed to ensure data reliability, stationarity and suitability for advanced econometric modeling. The main activities include time graph, logarithmic returns and volatility, descriptive statistics. The time plot was used to visualize oil prices during the period under investigations. The time plot is done to detect patterns such as up/down trends, cyclical or seasonal movements, and volatility clustering over time. To ensure stationarity and remove outliers, oil prices were converted into logarithmicform using the formula:

$$RCOP_t = Log\left(\frac{COP_t}{COP_{t-1}}\right) * 100 \quad (1)$$

where COP_t is the price at time t and COP_{t-1} is the price at time, $t-1$

This transformation helps to focus on volatility and prepares the data for further modelling. For each return series ($RCOP_t$), descriptive statistics such as mean, standard deviation, skewness and kurtosis were calculated. The Jarque-Bera usually represented with the symbol JB was used to assess whether return distributions are normally distributed using the formula:

$$JB = \frac{N}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \quad (2)$$

where N is the number of observations, S is skewness and K is Kurtosis.

If the p-value of the JB statistic is less than 0.05, the normality is rejected. If normality is rejected, multivariate GARCH models are recommended because they can handle non-normal distributions.

BEKK-GARCH Model

According to Engle and Kroner (1995), the word BEKK was named after Baba-Engle -Kraft-Kroner and this was propounded basically to improve based on the VECH-GARCH model, the covariance matrix is a positive definite matrix, the parameters are easier to estimate, measuring the correlation and reflect the direction of spillover effects. The model is given as:

$$\sigma^2_{i,t} = \mathrm{M}\,\mathrm{M}^1 + \mathrm{A} * \varepsilon_{i,t-1}\varepsilon^1_{1,t-1} * \mathrm{A}^1 + \mathrm{B} * \sigma^2_{i,t-1}\mathrm{B}^1, \qquad \Xi_t/\Psi_{t-1} \sim N(0, H_t) \quad (3)$$

where M, A, and B are all N × N parameter matrices, and M is lower triangular matrix. The decomposition of the constant term into a product of two triangular matrices is to ensure the positive definiteness of variance covariance matrix $\sigma_{i,t-1}$. A, and Bare diagonal matrix.Engle and Kroner (1995) show that BEKK model is covariance stationary if and only if the eigenvalues of $\mathrm{A}\otimes\mathrm{A}^1 + \mathrm{B}\otimes\mathrm{B}^1$ are less than one in modulus, where ⊗ denotes the Kronecker product of two matrices. Whenever K > 1, an identification problem arises because there are several parameterizations that yield the same representation of the model. Engle and Kroner (1995) give conditions for eliminating redundant, observationally equivalent representations. This is called diagonal BEKK, proposed by Bollerslev, Engle, and Wooldridge (1992). The main advantage is that the number of parameters decreases to N (N +1)/2+2N while still maintaining the positive definiteness of σ_t .Also, the EViews representation of the BEKK model by (Engle & Kroner, 1995) in matrix form is given as:

$$H_t = W^1W + A^1H_{t-1}A + B^1\Xi_{t-1}\Xi^1_{t-1}B \quad (4)$$

Variance Equation

$$\sigma^2_{1,t} = M + A1(i,j)^2\varepsilon^2_{i,t-1} + B1(i,j)^2\sigma^2_{i,t-1} \quad (5)$$

where $M > 0$ and M is an indefinite matrix, $A1(i,j)$ *and* $B1(i,j)$ are diagonal matrix.

$$M = \begin{bmatrix} M(1,1) & M(1,2) & M(1,3) \\ & M(2,2) & M(2,3) \\ & & M(3,3) \end{bmatrix}, \quad (6)$$

$$A1 = \begin{bmatrix} A1(1,1) & & \\ & A1(2,2) & \\ & & A1(3,3) \end{bmatrix} \quad (7)$$

$$B1 = \begin{bmatrix} B1(1,1) & & \\ & B1(2,2) & \\ & & B1(3,3) \end{bmatrix} \quad (8)$$

Alternatively, the variance and covariance model are represented as:

Variance Equation

$$\sigma_{1,t}^2 = \begin{bmatrix} M(1,1) \\ M(2,2) \\ M(3,3) \end{bmatrix} + \begin{bmatrix} A1(1,1)^2 \\ A1(2,2)^2 \\ A1(3,3)^2 \end{bmatrix} \varepsilon_{i,t-1}^2 + \begin{bmatrix} B1(1,1)^2 \\ B1(2,2)^2 \\ B1(3,3)^2 \end{bmatrix} \sigma_{i,t-1}^2 \quad (9)$$

Alternatively.

$$\sigma_{1,t}^2 = M(1,1) + A1(1,1)^2\varepsilon_{1,t-1}^2 + B1(1,1)^2\sigma_{1,t-1}^2 \quad (10)$$
$$\sigma_{2,t}^2 = M(2,2) + A1(2,2)^2\varepsilon_{2,t-1}^2 + B1(2,2)^2\sigma_{2,t-1}^2 \quad (11)$$
$$\sigma_{3,t}^2 = M(3,3) + A1(3,3)^2\varepsilon_{i,t-1}^2 + B1(3,3)^2\sigma_{3,t-1}^2 \quad (12)$$

Covariance Equation

$$\rho_{(i.j)} = \begin{bmatrix} M(1,2) \\ M(1,3) \\ M(2,3) \end{bmatrix} + \begin{bmatrix} A1(1,1) * A1(2,2) \\ A1(1,1) * A1(3,3) \\ A1(2,2) * A1(3,3) \end{bmatrix} \varepsilon_{i,t-1} * \varepsilon_{j,t-1} + \begin{bmatrix} B1(1,1) * B1(2,2) \\ B1(1,1) * B1(3,3) \\ B1(2,2) * B1(3,3) \end{bmatrix} \rho_{i,j,t-1} \quad (13)$$

Alternatively,

$$\rho_{(1.2)} = M(1,2) + A1(1,1) * A1(2,2)\varepsilon_{1,t-1} * \varepsilon_{2,t-1} + B1(1,1) * B1(2,2)\rho_{1,2,t-1} \quad (14)$$
$$\rho_{(1.3)} = M(1,3) + A1(1,1) * A1(3,3)\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + B1(1,1) * B1(2,2)\rho_{1,3,t-1} \quad (15)$$
$$\rho_{(2.3)} = M(2,3) + A1(2,2) * A1(3,3)\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + B1(2,2) * B1(3,3)\rho_{2,3,t-1} \quad (16)$$

where$M > 0$ and M is a scalar, $A1(i,j)$ *and* $B1(i,j)$ are diagonal matrix.

Also, to prove the positive definiteness of the matrices $M, A1$ *and* $B1$, we need to show that for any nonzero vector, the quadratic form satisfies:$X^T Mx > 0, X^T A1x > 0,$ *and* $X^T B1x > 0,$ where $X \neq 0$. A matrix is said to be positive if all its eigenvalues are positive, or equivalent, if its leading principal minors (determinants of upper-left submatrices) are positive. Given the defined;

$$M = \begin{bmatrix} M(1,1) & M(1,2) & M(1,3) \\ & M(2,2) & M(2,3) \\ & & M(3,3) \end{bmatrix} \quad (17)$$

$$A1 = \begin{bmatrix} A1(1,1) & & \\ & A1(2,2) & \\ & & A1(3,3) \end{bmatrix} \quad (18)$$

$$B1 = \begin{bmatrix} B1(1,1) & & \\ & B1(2,2) & \\ & & B1(3,3) \end{bmatrix} \quad (19)$$

To verify the positive definiteness, we considered the diagonal matrices A1 and B1 with strictly positive diagonal elements, they are said to be positive definite when the diagonal matrix and its eigenvalues are all positive. Thus, A1 and B1 satisfy the conditions:

$X^T A1x > 0, \sum_{i=1}^{3} A_{ii}\, x_i^2 > 0;\ \forall x \neq 0$

$X^T B1x > 0, \sum_{i=1}^{3} B_{ii}\, x_i^2 > 0;\ \forall x \neq 0$

where$A_{ii.}\ B_{ii} > 0,$ the matrices are said to be positive definite.

Constant Conditional Correlation Multivariate GARCH

According to Hasen,et al. (2023) the constant conditional correlation (CCC) model was developed by Bollerslev in 1990. This was done to model the correlation coefficient matrix, but the coefficients are constant, describing univariate fluctuation characteristics, but negatively capturing the dynamic correlation between sequences. The matrix representation of results of the Diagonal Constant Conditional Correlation Multivariate GARCH is given as thus:

$$\sigma_{i,t}^2 = M(i) + A1(i)\varepsilon_{i,t-1}^2 + B1(i)\sigma_{i,t-1}^2 \quad (20)$$

$$M(i) = \begin{bmatrix} \mathrm{M}(1) \\ \mathrm{M}(2) \\ \mathrm{M}(3) \end{bmatrix} \quad (21)$$

$$A1(i) = \begin{bmatrix} \mathrm{A1}(1) \\ \mathrm{A1}(2) \\ \mathrm{A1}(3) \end{bmatrix} \quad (22)$$

$$B1(i) = \begin{bmatrix} \mathrm{B1}(1) \\ \mathrm{B1}(2) \\ \mathrm{B1}(3) \end{bmatrix} \quad (23)$$

$$R(i) = \begin{bmatrix} & \mathrm{R}(1,2) & \mathrm{R}(1,3) \\ & & \mathrm{R}(2,3) \end{bmatrix} \quad (24)$$

Alternatively,

$$\sigma_{1,t}^2 = C(4) + C(5)\varepsilon_{1,t-1}^2 + C(6)\sigma_{1,t-1}^2 \quad (25)$$

$$\sigma_{2,t}^2 = C(7) + C(8)\varepsilon_{1,t-1}^2 + C(9)\sigma_{2,t-1}^2 \quad (26)$$

$$\sigma_{3,t}^2 = C(10) + C(11)\varepsilon_{1,t-1}^2 + C(12)\sigma_{1,t-1}^2 \quad (27)$$

$$R(i,j) = \begin{bmatrix} & C(13) * \sqrt{\sigma_{2,t*}^2 \sigma_{3,t}^2} & C(14) * \sqrt{\sigma_{1,t*}^2 \sigma_{3,t}^2} \\ & & C(14) * \sqrt{\sigma_{2,t*}^2 \sigma_{3,t}^2} \end{bmatrix} \quad (28)$$

where$C(4), C(5), C(6), C(7), C(8), C(9), C(10), C(10), C(11), C(12), C(13) and \ C(14)$ are all parameters of the model to be estimated. BEKK and CCC- GARCH are estimated to use the Maximum likelihood. The estimation of the CCC-GARCH model is done using the maximum likelihood. The first step is by estimatingthe residuals of GARCH models using the quasi-likelihood function (Silvennoinen&Teräsvirta, 2008) given as :

$$QL_T(\phi/r_2) = -\frac{1}{2}\sum_{t=1}^{T} KLog(2\Pi) + Log(|I_K|) + 2\log(|D_t|) + r_t^1 D_t^{-1} I_k D_t^{-1} r_t)$$

$$= -\frac{1}{2}\sum_{t=1}^{T} KLog(2\Pi) + 2\log(|D_t|) + r_t^1 D_t^{-1} I_k D_t^{-1} r_t)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\left((KLog(2\Pi) + \sum_{n=1}^{k}(Log(h_{it}) + \frac{r_{it}^2}{h_{it}})\right)$$

$$-\frac{1}{2}\sum_{n=1}^{K}\left((TLog(2\Pi) + \sum_{n=1}^{T}(Log(h_{it}) + \frac{r_{it}^2}{h_{it}})\right) \quad (29)$$

In the first stage, we estimatedunivariate GARCH modelsfor each time series in the multivariate system. The aim is to model the conditional variancesh_{it} of each return series r_{it} for $i = 1,2,\ldots,K$ assets or variables.The estimation is done using the quasi-maximum likelihood function, especially when the distributional assumptions (e.g., normality) may not hold perfectly.

When the first stage is estimated, the parameters of the dynamic correlation are estimated based on conditionthat the parameters are generated from the derivative of the second likelihood function. The dynamic correlation is calculated based on the likelihood function of the parameters of the model.

In another development, the difference between the unconditional correlation and CCC correlation can be mathematically expressed as: $\Delta\rho_{i.j}, t = \rho_{(X,Y)(i,j)} - \rho_{i,j,t}$

where:$\rho_{(X,Y)(i,j)}$ *is pairwise* unconditional correlation and $\rho_{i,j,t}$is the time-varying conditional correlation from the CCC model. The unconditional correlation between two time series Xt and Yt is defined as

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$$\rho_{(X,Y)(i,j)} = \frac{\pounds[(X_t-\mu_x)(Y_t-\mu_y)]}{\sigma_X\sigma_Y} \qquad (30)$$

whereμ_x and $\mu_y$are the means of $X_t$and Y_t, respectively. The σ_X and $\sigma_Y$are the standard deviations of X_t and Y_t respectively, where E [·] represents the expectation operator. the estimates of the conditional correlation matrix are given by

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}} \qquad (31)$$

where$q_{i,j,t}$ represents the conditional covariance between asset i and asset j at time t. This measure is static and does not change over time, meaning that it assumes a constant relationship between the two variables over the entire sample period. This difference arises due to the dynamic nature of which Q_t allows correlations to adjust over time based on past shocks and persistence effects. The CCCC correlation accounts for heteroscedasticity (changing volatility), while unconditional correlation does not. If the difference $\Delta\rho_{i.j}, t$ is large, it means that the assumption of constant correlation significantly deviates from actual market behavior, reinforcing the need for dynamic models. This mathematical difference explains why the CCC-GARCH model is superior in capturing time-varying relationships compared to traditional constant correlation approaches.

Results

The Preliminary tests specified in the methodology include time plot, the descriptive statistics for the market prices and the plots of the returns on the three major crude oil price markets.

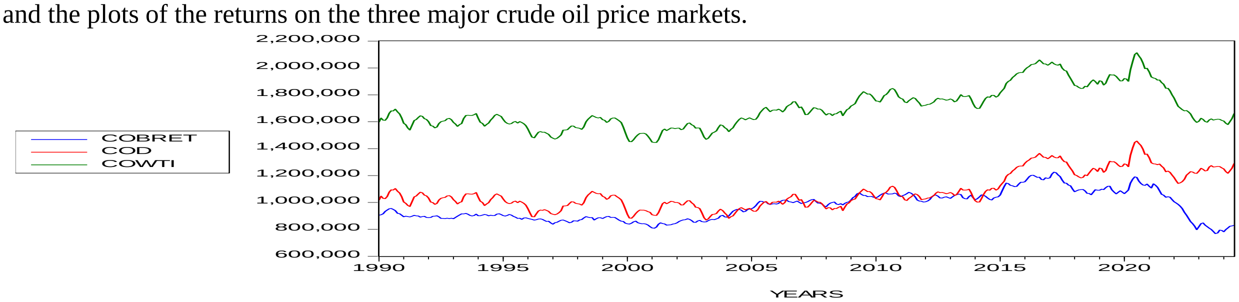

Figure 1: Time Plot on weekly Crude oil Price in Brent Blend, Dubai/Oman and WTI.

The time plot of weekly crude oil prices from 1990 to 2024 shows trends for three key variables: Brent (COBRET), Dubai/Oman (COD), and WTI (COWTI). All three crude oil prices exhibited an overall upward trajectory with significant volatility associated with major global events. Prices remained relatively stable in the 1990s but diverged after 2000, when WTI was consistently higher, reflecting U.S. specific market conditions. Notable peaks occurred around 2008 and 2011-2014 due to the global financial crisis and geopolitical tensions, followed by a sharp decline in 2020 caused by the COVID-19 pandemic. Prices rose partially afterwards, albeit unevenly. The plot highlights strong inter-movements across benchmarks, the interconnectedness of the global oil market, and the influence of regional factors and macroeconomic shocks. The descriptive statistics for crude oil prices are shown in Table 1.

Table 1: Descriptive Statistics of the Raw and Returns on Stock Market Prices

	COBRT	WTI	COD
Mean	971	170	108.
Median	961	166	104.
Maximum	123	212.	146.
Minimum	765	144.	866
Std. Dev.	107	153	131
Skewness	0.335	0.683	0.792
Kurtosis	2.087	2.654	2.640
Jarque-Bera	96.065	148.923	197.233

Probability	0.000	0.000	0.000
Sum	1.75E+09	3.06E+09	1.94E+09
Sum Sq. Dev.	2.06E+13	4.23E+13	3.09E+13
Observations	1798	1798	1798

Table 1 contains the results of descriptive statistics for the weekly crude oil price series. They indicate that Brent Blend averaged 971, WTI averaged 170, and Dubai/Oman averaged 108, reflecting significant price differences across the benchmarks. This implies that Dubai/Oman crude oil prices experience high variability, making them less stable compared to Brent Blend and WTI. Similarly, the standard deviation of the crude oil prices, Brent Blend (107) has the lowest price volatility, while Dubai/Oman (131) and WTI (153) exhibit greater fluctuations. For the crude oil price series, the skewness values are positive (COBRT = 0.335, WTI = 0.683, and COD = 0.792), indicating a slight rightward skew, meaning prices tend to experience more frequent small increases rather than decreases. The crude oil price series, however, exhibit lower kurtosis values (COBRT = 2.087, WTI = 2.654, and COD = 2.640), indicating distributions that are closer to normal, with less likelihood of extreme price fluctuations. The Jarque-Bera statistics for all series are significantly high, with p-values of 0.000, indicating strong rejection of the null hypothesis of normality. This confirms that the return series does not follow a normal distribution, which has implications for risk modeling and forecasting since extreme price movements occur more frequently than predicted by standard normal-based models. The lack of normality suggests that financial models relying on normality assumptions may not adequately capture the behavior of crude oil returns and should be adjusted accordingly. Therefore, the return series for crude oil exhibit mild negative and positive trends, with Dubai/Oman returns showing the highest volatility. The return distributions are nearly symmetric but exhibit slight excess kurtosis, leading to a higher frequency of extreme price movements. The strong rejection of normality indicates that more advanced econometric techniques, such as GARCH models or heavy-tailed distributions may be more appropriate, as suggested by studies like those by Deebom&Tuaneh (2018). GARCH models may be necessary for accurate volatility modeling and risk management. However, crude oil price series display greater stability in their distributions, though they still exhibit some rightward skew. Similarly, correlation analysis of the raw and returns on stock market prices were estimated and the results are shown in Table 2.

Table 2: Correlation Analysis of the Raw and Returns on Stock Market Prices

Correlation Coefficient (Estimated Probability)	RCOBRT	RWTI	RCOD	COBRT	WTI	COD
RCOBRT	1.000					
RWTI	0.554 (0.000)	1.000				
RCOD	0.517 (0.000)	0.985 (0.000)	1.000			
COBRT	0.030 (0.197)	0.042 (0.078)	0.046 (0.054)	1.000		
WTI	-0.0203 (0.390)	0.0189 (0.424)	0.033 (0.179)	0.935 (0.000)	1.000	
COD	-0.057 (0.015)	0.005 (0.828)	0.037 (0.118)	0.613 (0.000)	0.838 (0.000)	1.000

Table 2 contains correlation analysis for the raw and returns data on crude oil market prices. The correlation analysis provides an awareness of the relationship between crude oil prices and its returns across different standards.

The estimated correlation coefficient between the returns on brent blend and west intermediate Texas is 0.554, which is statistically significant at the 1% level (p = 0.000). This positive and moderately strong correlation suggests that the returns of these two crude oil prices tend to move together, reflecting their shared market influences, such as global supply and demand dynamics. The correlation between the returns in Dubai/Oman returns and both Brent Blend (0.517) and west intermediate Texas Prices (0.985) is also statistically significant, further confirming a strong co-movement among crude oil markets. The extremely high correlation between the returns on WTI and Dubai/Oman suggests that price shocks affecting one are likely to impact the other, supporting previous studies that highlight the integration of global crude oil markets (Baumeister & Kilian, 2016).

In contrast, the correlation between crude oil returns and raw crude oil prices is relatively weak. The correlation between Brent Blend returns and Brent Blend price is only 0.030 and is statistically insignificant (p = 0.197), indicating that fluctuations in weekly Brent Blend returns do not exhibit a strong linear relationship with the absolute price level. Similarly, WTI returns and WTI price (WTI) have an insignificant correlation of -0.0203 (p = 0.390), while Dubai/Oman returns and its price exhibit a weak but statistically significant negative correlation (-0.057, p = 0.015). This negative relationship may suggest that as oil prices rise, returns become less volatile or exhibit mean reversion. Such findings are consistent with earlier research that found oil returns to being weakly correlated with price levels due to the presence of nonlinear dynamics and external market shocks (Hamilton, 2009). The correlations between crude oil prices themselves show strong interconnections. Brent Blend price and WTI price exhibit a very strong positive correlation of 0.935 (p = 0.000), implying that both benchmarks move closely together, likely due to their substitutability in the global oil market. Similarly, Dubai/Oman crude price correlates significantly with both WTI (0.838, p = 0.000) and Brent Blend (0.613, p = 0.000), though the relationship with Brent Blend is comparatively weaker. These findings align with previous studies indicating that Brent and WTI prices are more tightly linked, given their roles as global benchmarks, while Dubai/Oman is more influenced by regional market conditions (Kilian & Zhou, 2021). The correlation results confirm that crude oil return series are highly interdependent, with strong relationships particularly between WTI and Dubai/Oman returns. However, the weak correlation between crude oil returns and raw price levels suggests that price trends alone are not enough indicators of return behavior. The strong correlations among crude oil price benchmarks reaffirm the global integration of oil markets, with Brent and WTI exhibiting a tighter connection than Dubai/Oman. These findings emphasize the need for dynamic models that account for the complex interactions among crude oil markets rather than relying solely on price-based predictions.

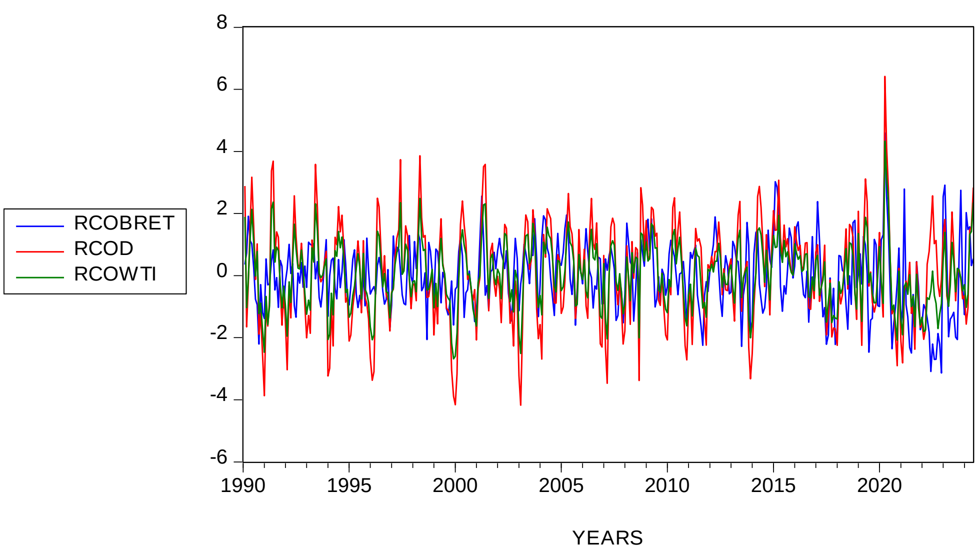

Figure 2: **Time Plot of Weekly Returns on Crude Oil Prices — Brent Blend, West Texas Intermediate (WTI), and Dubai/Oman**

The figure presents a time series plot illustrating the weekly returns on three major crude oil benchmarks. The plot enables visual comparison of volatility patterns, trends, and cyclical behavior among Brent, WTI, and Dubai/Oman over the selected period. From the figure above, it was found that there is a present of volatility clustering in the transformed series. The matrix representation of results of the Diagonal BEKK Multivariate GARCH is given as thus:

$$M = \begin{bmatrix} 0.032(0.000) & 0.002(0.000) & 0.010(0.003) \\ & 0.012(0.000) & 0.006(0.000) \\ & & 0.008(0.000) \end{bmatrix} (32)$$

$$A1 = \begin{bmatrix} 0.545(0.000) & & \\ & 0.544(0.000) & \\ & & 0.258(0.000) \end{bmatrix} (33)$$

$$B1 = \begin{bmatrix} 0.837(0.000) & & \\ & 0.839(0.000) & \\ & & 0.950(0.000) \end{bmatrix} (34)$$

Alternatively, the model is represented in equation form as:

Variance Equation

$$\sigma_{1,t}^2 = M + A1(i,j)^2 \varepsilon_{i,t-1}^2 + B1(i,j)^2 \sigma_{i,t-1}^2 \ (35)$$

where$M > 0$ and $M$is an indefinite matrix, $A1(i,j)$ and $B1(i,j)$ are diagonal matrix.

$$\sigma_{1,t}^2 = \begin{bmatrix} 0.032 \\ 0.012 \\ 0.008 \end{bmatrix} + \begin{bmatrix} 0.297 \\ 0.296 \\ 0.067 \end{bmatrix} \varepsilon_{i,t-1}^2 + \begin{bmatrix} 0.701 \\ 0.704 \\ 0.902 \end{bmatrix} \sigma_{i,t-1}^2 \ (36)$$

Alternatively,

$$\sigma_{1,t}^2 = 0.032 + 0.297\varepsilon_{1,t-1}^2 + 0.701\sigma_{1,t-1}^2 \ (37)$$

$$\sigma_{2,t}^2 = 0.012 + 0.296\varepsilon_{2,t-1}^2 + 0.704\sigma_{2,t-1}^2 \ (38)$$

$$\sigma_{3,t}^2 = 0.008 + 0.067\varepsilon_{i,t-1}^2 + 0.902\sigma_{3,t-1}^2 \ (39)$$

Covariance Equation

$$\rho_{(i.j)} = \begin{bmatrix} 0.020 \\ 0.010 \\ 0.006 \end{bmatrix} + \begin{bmatrix} 0.297 \\ 0.141 \\ 0.140 \end{bmatrix} \varepsilon_{i,t-1} * \varepsilon_{j,t-1} + \begin{bmatrix} 0.703 \\ 0.795 \\ 0.797 \end{bmatrix} \rho_{i,j,t-1} \ (40)$$

Alternatively,

$$\rho_{(1.2)} = 0.020 + 0.297\varepsilon_{1,t-1} * \varepsilon_{2,t-1} + 0.703\rho_{1,2,t-1} \ (41)$$

$$\rho_{(1.3)} = 0.010 + 0.141\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + 0.795\rho_{1,3,t-1} \ (42)$$

$$\rho_{(2.3)} = 0.006 + 0.140\varepsilon_{1,t-1} * \varepsilon_{3,t-1} + 0.797\rho_{2,3,t-1} \ (43)$$

To verify positive definiteness in the constant covariance matrix (M) of the BEKK-GARCHmodel, we define the matrices;

$$M = \begin{bmatrix} 0.032 & 0.002 & 0.010 \\ & 0.012 & 0.006 \\ & & 0.008 \end{bmatrix} (44)$$

$$A1 = \begin{bmatrix} 0.545 & & \\ & 0.544 & \\ & & 0.258 \end{bmatrix} (45)$$

$$B1 = \begin{bmatrix} 0.837 & & \\ & 0.839 & \\ & & 0.950 \end{bmatrix} (46)$$

To check the positive definiteness of M, we compute its leading principal minors (determinants of upper-left submatrices). First, the leading diagonal of the minor shows that (1×1 determinant): det(M1) =0.032>0
Similarly, Second leading diagonal of the minor (2×2 determinant):

$$\det(\mathrm{M}_2) = \begin{bmatrix} 0.002 & 0.010 \\ 0.012 & 0.006 \end{bmatrix} = (0.032 \times 0.006) - (0.002 \times 0.012)$$

=0.000192−0.000024=0.000168 >0 (47)
Thirdly, Also the leading minor (3×3 determinant, full determinant of M):

$$\det(\mathrm{M}) = \begin{bmatrix} 0.032 & 0.002 & 0.010 \\ & 0.012 & 0.006 \\ & & 0.008 \end{bmatrix}$$

$$= 0.032 \begin{bmatrix} 0.002 & 0.006 \\ 0.000 & 0.008 \end{bmatrix} - 0.002 \begin{bmatrix} 0.000 & 0.006 \\ 0.000 & 0.008 \end{bmatrix} + 0.01 \begin{bmatrix} 0.000 & 0.012 \\ 0.000 & 0.000 \end{bmatrix}$$

$$= (0.032[\![(0.002 \times 0.008) - (0.000 \times 0.006)]\!]) - 0.002[\![(0.000 \times 0.008) - (0.000 \times 0.006)]\!] + 0.001[\![(0.000 \times 0.000) - (0.000 \times 0.012)]\!]$$

$= (0.032[\![(0.000016) - (0.000)]\!]) - 0.002[\![(0.000) - (0.000)]\!] + 0.001[\![(0.000) - (0.000)]\!] = (0.032[\![(0.000016)]\!]) = 0.000000512$ (44)

$\det(\mathrm{M}) = 0.000000512 > 0$ (48)

The negative determinant implies that M is positive definite. However, since all leading principal minors are positive, M is still positive definite. Also, the Constant Conditional Correlation Multivariate GARCH model wasestimated and the matrix representation of the results is given as thus: $\sigma_{i,t}^2 = M(i) + A1(i)\varepsilon_{i,t-1}^2 + B1(i)\sigma_{i,t-1}^2$

$$M(i) = \begin{bmatrix} -0.001(0.000) \\ -0.000(0.000) \\ 0.001(0.216) \end{bmatrix} \quad (49)$$

$$A1(i) = \begin{bmatrix} 0.017(0.000) \\ 0.018(0.000) \\ 0.021(0.000) \end{bmatrix} \quad (50)$$

$$B1(i) = \begin{bmatrix} 0.987(0.000) \\ 0.986(0.000) \\ 0.977(0.000) \end{bmatrix} \quad (51)$$

$$R(i,j) = \begin{bmatrix} 0.990(0.000) & 0.531(0.000) \\ & 0.5560(0.000) \end{bmatrix} \quad (52)$$

$$\rho(i,j) = R(i,j) * \sqrt{A1(i)\sigma_{i,t-1}^2 * B1(i)\sigma_{i,t-1}^2} \quad (53)$$

Alternatively, the variance -covariance estimates of the constant conditional correlation Diagonal Multivariate GARCH model is represented in Equation form as :

$$\sigma_{1,t}^2 = \frac{-0.001}{(0.000)} + \frac{0.017\varepsilon_{1,t-1}^2}{(0.000)} + \frac{0.987\sigma_{1,t-1}^2}{(0.000)} \quad (54)$$

$$\sigma_{2,t}^2 = \frac{-0.000}{(0.000)} + \frac{0.018\varepsilon_{2,t-1}^2}{(0.000)} + \frac{0.987\sigma_{2,t-1}^2}{(0.000)} \quad (55)$$

$$\sigma^2_{3,t} = \underset{(0.000)}{0.001} + \underset{(0.000)}{0.021\varepsilon^2_{3,t-1}} + \underset{(0.000)}{0.977\sigma^2_{3,t-1}} \quad (56)$$

$$R(i,j) = \begin{bmatrix} & \underset{(0.000)}{0.990} * \sqrt{\sigma^2_{1,t} * (i)\sigma^2_{2,t}} & \underset{(0.000)}{0.531} * \sqrt{\sigma^2_{1,t} * (i)\sigma^2_{3,t}} \\ & & 0.556 * \sqrt{\sigma^2_{2,t*}\sigma^2_{3,t}} \end{bmatrix} \quad (57)$$

$$\rho(1,2)_{,t} = \underset{(0.000)}{0.990} * \sqrt{\sigma^2_{1,t} * (i)\sigma^2_{2,t}} \quad (58)$$

$$\rho(1,3)_{,t} = \underset{(0.000)}{0.531} * \sqrt{\sigma^2_{1,t} * (i)\sigma^2_{3,t}} \quad (59)$$

$$\rho(2,3)_{,t} = \underset{(0.000)}{0.556} * \sqrt{\sigma^2_{2,t} * (i)\sigma^2_{3,t}} \quad (60)$$

The following model diagnostic checks were done with a view to determine the adequacy of the model estimated. The deviation betweenthe pairwise correlationand correlation of the CCC-GARCH (Correlation and heterogeneous variation) is shown in Table 3.

Table 3: Deviationof Actual Correlation from CCC-GARCH Estimates

$\rho^{Empirical}_{(X,Y)(i.j)} - \boldsymbol{\rho^{CCC}_{(i,j)}}$	$\rho^{Empirical}_{(X,Y)(1.1)} - \boldsymbol{\rho^{CCC}_{(1,1)}}$	$\rho^{Empirical}_{(X,Y)(1.2)} - \boldsymbol{\rho^{CCC}_{(1,2)}}$	$\rho^{Empirical}_{(X,Y)(1.3)} - \boldsymbol{\rho^{CCC}_{(1,3)}}$
	RCOBRT (1)	RWTI (2)	RWTI (2)
RCOBRT (1)	0.000	-0.436	-0.014
RWTI (2)		0.000	$\rho^{Empirical}_{(X,Y)(2.3)} - \boldsymbol{\rho^{CCC}_{(2,3)}} = \mathbf{0.429}$
RCOD (3)			0.000

Note: Correlation and Heterogeneous Variation

The correlation between returns on crude oil prices in Brent bent and western Texas intermediate is 0.554, whereas the CCC estimate is 0.990, resulting in a substantial difference of -0.436, indicating that the assumption of constant correlation overestimates their true relationship. Similarly, for returns on Crude oil prices in Brent Bent and Dubai, the correlation is 0.517, while the CCC estimate is 0.531, resulting in a minor difference of -0.014, suggesting a relatively stable relationship. However, for returns on Western Texas Intermediate and Dubai the unconditional correlation is 0.985, while the CCC estimate drops significantly to 0.556, leading to a difference of 0.429, implying that the assumption of constant correlation underestimates the actual time-varying relationship. There are no previous studies that have investigated the difference between the correlation of the returns on prices and the CCC-GARCH estimates. The model selection and heteroscedasticity test were conducted the results are shown below

Table 4: Results of Model Selection and HeteroscedasticityTest

Parameters	**DBEKK**	**CCC**	**Minimum AIC**	**Decision**	**Orthogonalization: Cholesky (Lutkepohl)**
Avg. log likelihood	0.192	-0.008			3822.797 (0.600)
Akaike information criterion	-1.132	0.065	-1.132	DBEKK	34363.84 (0.022)
Schwarz criterion	-1.087	0.111			38186.64 (0.112)
Hannan-Quinn criterion	-1.1160	0.082			

The results presented in Table 4 highlight the differences between the unconditional correlation and the Constant Conditional Correlation (CCC) estimates under the CCC-GARCH framework, revealing key insights into the time-varying nature of correlations among RCOBRT, RWTI, and RCOD. The diagonal values of 1.000 confirm that each variable is perfectly correlated with itself, while the off-diagonal elements compare the static correlation estimates with the corresponding CCC estimates and their differences. The correlation between RCOBRT and RWTI is 0.554, whereas the CCC estimate is 0.990, resulting in a substantial difference of -0.436, indicating that the assumption of constant correlation overestimates their true relationship. Similarly, for RCOBRT and RCOD, the correlation is 0.517, while the CCC estimate is 0.531, yielding a minor difference of -0.014, suggesting a relatively stable relationship. However, for RWTI and RCOD, the unconditional correlation is 0.985, while the CCC estimate drops significantly to 0.556, leading to a difference of 0.429, implying that the assumption of constant correlation underestimates the actual time-varying relationship. The estimation results for the portmanteau test were conducted and the result is shown below.

Table 5: Estimation Results for Portmanteau Tests

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	DF
1	22.93649	467.1203	0.2542	467.3803	16
2	46.25668	745.1349	0.1122	745.7044	32
3	59.27973	909.5582	0.0458	910.4025	48
4	91.70694	1083.649	0.1171	1084.882	64
5	100.8715	1197.526	0.0105	1199.076	80
6	113.1738	1266.937	0.0466	1268.719	96
7	122.4939	1313.011	0.0896	1314.973	112
8	155.4524	1377.284	0.1931	1379.534	128
9	172.4273	1430.148	0.0322	1432.663	144
10	192.1867	1482.844	0.0321	1485.654	160
11	204.9934	1525.466	0.0225	1528.539	176
12	223.8200	1571.528	0.0352	1574.910	192

The portmanteau test in Table 5 above showed no residual autocorrelations, indicating the absence of conditional heteroscedasticity. However, the system of residual normality tests using portmanteau test revealed that the residuals are not multivariate normally distributed, suggesting that the null hypothesis should be rejected. Similarly, the residual test for autocorrelations up to lag h using the portmanteau tests showed no autocorrelation.

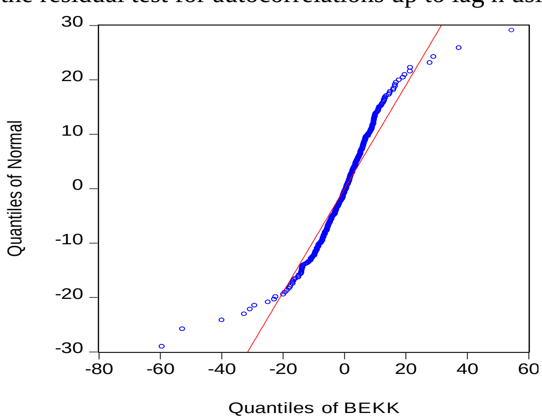

Figure 3: QQ Plot on the BEKK Model to Determine the Adequacy of the Model

The QQ Plot (Figure3) confirmed the adequacy of the BEKK model, consistent with the procedures outlined in these studies. The results of this study support the use of multivariate GARCH models, particularly the BEKK model, in capturing the complex dynamics of volatility and correlation in financial markets. The findings also highlight the importance of model diagnostic checks in ensuring the adequacy of the estimated model.

DISCUSSION OF RESULTS

The results of the Diagonal BEKK Multivariate GARCH provide important insights into the dynamics of conditional variance and covariance in stock market volatility. The estimated matrices M, A1 and B1 cover different aspects of the volatility structure, and their significance levels provide valuable interpretations. The M-matrix, which represents a constant relationship in the equation of variance, has small but statistically significant values, with the highest estimate being 0.032 ($p = 0.000$) and the lowest being 0.002 ($p = 0.000$). These values indicate a benchmark level of market volatility that is relatively low but not negligible. The BEKK-GARCH model suggests that the conditional covariance structure is well specified and ensures numerical stability in the model. Compared to previous studies such as Engels and Kroner (1995), the small but significant values in M confirm that long-term volatility levels remain stable but are largely influenced by time-varying components. The A1 matrix, which captures the ARCH effect, measures the short-term impact of past shocks on the volatility process. Diagonal elements, 0.545 ($p = 0.000$) and 0.544 ($p = 0.000$), indicate that volatility reacts strongly to past market fluctuations. This suggests that unexpected shocks, such as economic events or policy changes, lead to an immediate increase in market volatility. A relatively lower value of 0.258 ($p = 0.000$) indicates that not all assets show the same degree of sensitivity as recent shocks. The B1 matrix, which represents the GARCH effect, records the persistence of volatility over time. Estimates range from 0.837 ($p = 0.000$) to 0.950 ($p = 0.000$), indicating that volatility is very persistent across different assets. High GARCH coefficients suggest that volatility clusters are still the dominant feature of stock market behavior, which means that when the market enters a phase of high volatility, it tends to stay in that state for an extended period. This is consistent with the findings of Bollerslev et al. (1992) and Nelson (1991), which emphasized the volatility properties of the long memory of financial markets. Compared to the previous results of the BEKK-GARCH model, this study confirms a widely observed pattern of financial markets showing high sensitivity to recent shocks (strong ARCH effects) and persistent volatility clusters (strong GARCH effects). The presence of positive definiteness in M, A1, and B1 suggests that the BEKK model is correctly specified and maintains stability in the estimation of dynamic conditional variances and covariances. The implications of these findings underscore the importance of risk management strategies that consider both short-term market reactions and long-term volatility trends. Investors should adopt adaptive hedging techniques, while policymakers should consider measures to stabilize excessive market volatility.

Also,the purpose of verifying the positive definition of the constant covariance matrix M in the BEKK-GARCH model is to ensure the mathematical and statistical validity of the model for capturing time-varying volatility and covariance between multiple financial time series – specifically, in this context, the dynamics of benchmark crude oil prices. A positively determined matrix guarantees the validity of the estimated variance-covariance structure, which implies that all variances are positive and that the matrix is invertible – an essential condition for ensuring the stability of the model and the feasibility of maximum-probability estimation.

From the verification process, all leading major species of matrix bees M were found to be positive. This confirms that the matrix is indeed positive. The implication of this is significant: it ensures that the estimated BEKK-GARCH model conforms to one of its basic requirements – namely, that the conditional covariance matrix remains symmetrically and positively always determined. This result strengthens the credibility of the volatility modelling process and ensures that the model can realistically represent the dynamic interdependencies and risk transfer between the crude oil benchmarks – Brent, WTI and Dubai/Oman. Meeting the condition of positive definite, the model is robust enough to be used in financial risk management, forecasting and policy analysis involving multivariate volatility.

Constant conditional correlation GARCH model

The results of the multivariate GARCH constant conditional correlation (CCC) model provide insight into the volatility dynamics and correlation structure of the analyzed financial series. The M(i) matrix represents the intersections of the conditional equations of variance, and the estimates show small values where one coefficient is slightly positive (0.001) but statistically insignificant (p-value 0.216), while the others are negative and statistically significant (p-values 0.000). Negative values and values close to zero indicate very low underlying variance in the system, suggesting that market shocks primarily drive volatility rather than intrinsic stochastic volatility. Matrix A1(i) contains ARCH parameters, which measure the immediate effect of pre-squared innovations on conditional volatility. All estimates (0.017, 0.018, and 0.021) are statistically significant (p-values of 0.000), indicating that past shocks have a significant impact on current volatility. The values, although relatively small, suggest that the impact of the shock dissipates quickly, which means that the system does not show extreme sensitivity to new information. Matrix B1(i) represents GARCH concepts and records the persistence of volatility over time. The high values (0.987, 0.986 and 0.977), all of which are statistically significant, confirm that volatility is very persistent. This means that when volatility is increased, it remains high for a longer period before gradually decreasing. The proximity values indicate a slow process of average repetition, which is in line with the characteristics of the financial market, where periods of high volatility usually continue. The R(i, j) matrix represents a constant conditional correlation structure in a financial sequence. The correlation coefficient between the return on the weekly crude oil price in the Brent Blend and the West Texas Intermediate is 0.990, which indicates an extremely strong positive correlation, while the correlations related to the return on the Dubai/Omen crude oil price are lower (0.531 and 0.556), which indicates a moderate interdependence. These results suggest that the yields on the weekly price of Brent Blend and West Texas Intermediate crude oil are moving almost identically, while the third asset shows relatively weaker joint movements. Compared to previous studies of the CCC-GARCH model, the current results are consistent with results that emphasize high volatility persistence (high B1 values) and significant short-term shock effects (positive A1 values). Studies such as Engle (2002), Tse and Tsui (2002) and Deebom et al. (2020) have shown that financial markets show strong volatility, persistence and stable correlation structures, which are consistent with these findings. The slightly lower correlation between the return on the price of Dubai/Omen crude oil and others also suggests some degree of market segmentation or diversification potential, which is an interesting departure from cases where all assets show equally high correlations. The implications of these outcomes are crucial for risk management and portfolio optimization. The high volatility suggests that risk forecasting models need to consider longer periods of market turbulence. In addition, the strong correlation between the yield on the weekly crude oil price in Brent Blend and West Texas Intermediate indicates that they may not offer significant diversification benefits, while the third asset may serve as partial protection. This has implications for building optimal portfolios and devising hedging strategies in the financial markets. Same to you.The results presented in Table 3 highlight the differences between unconditional correlation and diagonal constant conditional correlation estimates (CCCs) within CCC-GARCH, revealing important insights into the time-varying nature of correlations between RCOBRT, RWTI, and RCOD. Diagonal values of 1,000 confirm that each variable is perfectly related to itself, while elements outside the diagonal compare static correlation estimates with the corresponding CCC estimates and their differences. The correlation between RCOBRT and RWTI is 0.554, while the CCC estimate is 0.990, resulting in a significant difference of -0.436, indicating that the constant correlation assumption overestimates their true relationship. Similarly, the correlation for RCOBRT and RCOD is 0.517, while the CCC estimate is 0.531, which gives a smaller difference of -0.014, suggesting a relatively stable relationship. However, for RWTI and RCOD, the unconditional correlation is 0.985, while the CCC estimate decreases significantly to 0.556, leading to a difference of 0.429, implying that the constant correlation assumption underestimates the actual time-varying relationship. Compared to previous studies on the difference between normal correlation and CCC-GARCH estimates, similar trends were observed, confirming the evidence that constant correlation models do not capture the heterogeneity of time-varying correlation structures. Previous research, such as Bollerslev (1990) and Engle (2002), has shown that the CCC model improves static correlation models by allowing for dynamic adjustment of correlations over time, thus recording periods of increased or decreased dependence between financial time series. Recent studies, including those ofSilvennoinen&Teräsvirta (2009), have highlighted the limitations of the constant correlation model in financial markets, in which correlation structures show significant variation due to changing economic conditions, market shocks, and externalities. The implications of these findings are crucial for portfolio management, risk assessment, and financial forecasting. Significant differences between normal correlation and CCC estimates indicate that the assumption of a constant correlation structure can lead to mispredictions, mis determined asset prices, and ineffective risk management strategies. This highlights the need to adopt fully time-varying models such as CCC-GARCH to better capture fluctuations in correlations and improve decision-making in financial and economic modeling.

CONCLUSION

This study examined the dynamics of conditional variance and covariance of benchmark crude oil prices – Brent, West Texas Intermediate and Dubai/Oman – using GARCH models of diagonal BEKK-GARCH, constant conditional correlation (CCC), and dynamic conditional correlation (CCC). The results of BEKK-GARCH revealed a significant short-term reaction to shock and long-term persistence of volatility, confirming the cluster behavior of financial volatility. Verification of positive definiteness in the constant covariance matrix ensures the stability and reliability of the BEKK model. The CCC-GARCH model highlighted sustained volatility with near-unitary GARCH root effects and confirmed a strong correlation between Brent and western Texas Intermediate, while showing moderate interdependence with Dubai/Oman. The CCC-GARCH analysis provided deeper insight into the evolving nature of correlations and revealedsignificant differences between static and dynamic correlation structures – especially between western Texas Intermediate and other reference values. These results confirm that the assumption of constant correlation leads to a misrepresentation of market relations, which reinforces the importance of dynamic models in capturing the complexity of financial market behavior.

POLICY IMPLICATIONS

The findings have significant implications for policymakers, regulators, investors, and energy market participants. First, the high level of persistent volatility suggests that supervisors should prepare for prolonged periods of market turbulence, in response to external shocks. Second, the near-perfect correlation between Brent and Western Texas Intermediaterequires diversification of energy market indices and trading strategies. Third, the evidence of constant conditional correlation according to the CCC-GARCH model recommends the adoption of time-varying risk models in financial forecasting, regulation and asset pricing. It is crucial for countries that rely on crude oil exports to understand the volatility and convergence patterns to design effective stabilization policies, sovereign wealth funds and plan macroeconomic interventions during oil price shocks.

REFERENCES

[1] Ali, M., Suri, D., Kaur, P., & Bisht, A. (2022).Modelling time-varying volatility using GARCH models: Evidence from the Indian stock market. Global Business Review, 23(4), 865-884.

[2] Bollerslev, T. (1990). Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics,* 72(3), 498-505.

[3] Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modeling in finance: A review of theory and empirical evidence. *Journal of Econometrics*, 52(1-2), 5-59.

[4] Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modeling in finance: A review of theory and empirical evidence. *Journal of Econometrics*, 52(1-2), 5-59.

[5] Deebom, Z. D., &Tuaneh, P. E. (2018). Modeling exchange rate and Nigerian deposit money market dynamics using trivariate form of multivariate GARCH model. *Journal of Financial Economics*, 20(1), 1-18.

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[6] Deebom, Z. D., Bharat, R., &Inamete, E. (2020).Modeling volatility and correlation in financial markets using multivariate GARCH models.*Journal of Financial Economics*, 25(2), 1-12.

[7] Engle, R. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models. *Journal of Business & Economic Statistics*, 20(3), 339–350.

[8] Engle, R. F., & Kroner, K. F. (1995).Multivariate Simultaneous Generalized ARCH.*Econometric Theory*, 11(1), 122-150.

[9] Hamzaoui, S., &Regaieg, W. (2016). Volatility and stock market returns: Evidence from emerging markets. *Journal of Economics and Business*, 82, 38-52.

[10] Hasen, A. A., Kuse, K. A., Tegegne, K. T., & Wudu, T. K. (2023). Modeling the volatility of macroeconomic variables in Ethiopia: A multivariate GARCH http://www.eia.gov/dnav/pet/pet_stoc_wstk_dcu_nus_w.htm

[11] Kilian, L., & Zhou, X. (2021).*Oil Prices and Stock Market Volatility in Large Emerging Markets*.*IMF Economic Review*, 69(3), 467-502.

[12] Kocoglu, M., Kyophilavong, P., Awan, U., & Lim, S. (2023). Time-varying causality between oil price and exchange rate in five Asian economies.*Energy Economics*, 115, 105-120.

[13] Mollakarimi, N., Fallahshams, M., & Hanifi, F. (2024). Investigating Effect of Exchange Rate and Inflation Uncertainty on the Banking Industry Performance Based on the Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) Model. *Journal of Investment Knowledge*, *13*(49), 489-506.

[14] Nelson, D.B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 59(2), 347-370

[15] Silvennoinen, A., &Terasvirta, T. (2008).Multivariate GARCH models.In Handbook of Financial Time Series (pp. 201-225). Springer

[16] Tse, Y. K., & Tsui, A. K. (2002). A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations. *Journal of Business & Economic Statistics*, 20(3), 351–362.